METHODS FOR REDUCING HEAT LOSSES FROM FLAT PLATE SOLAR COLLECTORS. PHASE II

Final Report, February 1, 1976—August 31, 1977

By
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March 1978

Work Performed Under Contract No. EY-76-G-02-2597

University of Waterloo
Waterloo, Ontario, Canada

U.S. Department of Energy

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METHODS FOR REDUCING HEAT LOSSES FROM FLAT PLATE SOLAR COLLECTORS

PHASE II

Final Report
for Period Feb. 1, 1976-Aug. 31, 1977

WRI Project No. 510-08

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March, 1978

Prepared for
THE U.S. DEPARTMENT OF ENERGY
UNDER CONTRACT NO. EY-76-C-02-2597.*000 (ERDA)

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ABSTRACT

Improvements to flat plate solar collectors for heating and cooling of buildings were investigated through two parallel studies. The first study, which deals with the free convective heat loss from V-corrugated absorber plate to a plane glass cover, has shown that, for the same average spacing, the free convective heat loss is greater for a V-corrugated absorber plate than for a plane absorber plate. However, provided the average spacing is large enough, the amount of increase is slight. The second study, which deals with the free convective heat loss in a honeycomb solar collector in which the honeycomb consists of a set of horizontal partitions, or slits, has shown that provided the solar collector is tilted to near vertical, such a honeycomb gives equivalent or superior free convective loss suppression than does a square-celled honeycomb having the same amount of material. Correlation equations for the free convective heat loss are given for both studies.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ABSTRACT</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>1. SUMMARY</td>
<td>1</td>
</tr>
<tr>
<td>2. INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>3. FREE CONVECTION ACROSS INCLINED AIR LAYERS BOUNDED BY ONE FLAT SURFACE AND ONE V-CORRUGATED SURFACE</td>
<td>7</td>
</tr>
<tr>
<td>3.1 Description of Problem</td>
<td>7</td>
</tr>
<tr>
<td>3.2 Previous Studies</td>
<td>9</td>
</tr>
<tr>
<td>3.3 Description of Experimental Apparatus</td>
<td>10</td>
</tr>
<tr>
<td>3.4 Dimensional Analysis</td>
<td>11</td>
</tr>
<tr>
<td>3.5 Range of Measurements</td>
<td>13</td>
</tr>
<tr>
<td>3.6 Evaluation of the Nusselt Number at Low Rayleigh Number - the Conduction Limit</td>
<td>14</td>
</tr>
<tr>
<td>3.7 Evaluation of Radiative Heat Transfer Correction</td>
<td>16</td>
</tr>
<tr>
<td>3.8 Experimental Results</td>
<td>17</td>
</tr>
<tr>
<td>3.9 Discussion of Results</td>
<td>22</td>
</tr>
<tr>
<td>3.10 Correlation Equation</td>
<td>22</td>
</tr>
<tr>
<td>3.11 Application to the Design of Solar Collectors</td>
<td>27</td>
</tr>
<tr>
<td>4. FREE CONVECTION ACROSS AIR LAYERS CONSTRAINED BY A HONEYCOMB OF LARGE PLAN ASPECT RATIO</td>
<td>29</td>
</tr>
<tr>
<td>4.1 Problem Background and Description</td>
<td>29</td>
</tr>
<tr>
<td>4.1.1 Geometry</td>
<td>29</td>
</tr>
<tr>
<td>4.1.2 Radiation Aspects</td>
<td>32</td>
</tr>
<tr>
<td>4.2 Previous Studies</td>
<td>33</td>
</tr>
<tr>
<td>4.3 Description of Apparatus</td>
<td>35</td>
</tr>
<tr>
<td>4.4 Dimensional Analysis and Range of Measurements</td>
<td>36</td>
</tr>
<tr>
<td>4.5 Results</td>
<td>40</td>
</tr>
<tr>
<td>4.6 Correlation Equations</td>
<td>51</td>
</tr>
<tr>
<td>4.6.1 Critical Rayleigh Numbers, $\theta &lt; 75^\circ$</td>
<td>51</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.6.2 Correlation Equation for $\theta \leq 75^\circ$</td>
<td>54</td>
</tr>
<tr>
<td>4.6.3 Correlation Equation for $\theta = 90^\circ$</td>
<td>55</td>
</tr>
<tr>
<td>4.7 Application to Design of Honeycomb Collectors</td>
<td>60</td>
</tr>
<tr>
<td>4.8 A Note on Radiative Transfer Across Honeycombs</td>
<td>63</td>
</tr>
<tr>
<td>5. CONCLUSIONS</td>
<td>60</td>
</tr>
<tr>
<td>6. LIST OF REFERENCES</td>
<td>70</td>
</tr>
<tr>
<td>7. NOMENCLATURE</td>
<td>74</td>
</tr>
<tr>
<td>8. LIST OF CONTRIBUTING PERSONNEL</td>
<td>77</td>
</tr>
<tr>
<td>9. CONTRACT COMPLIANCE</td>
<td>78</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>3-1</td>
<td>Sketch of the inclined air layers bounded by a V-corrugated plate and a flat plate. In configuration 'A' (upper sketch) the lower plate is V-corrugated and the upper is flat while in configuration 'B' (lower sketch) the reverse is the case. Both configurations were studied in the present study.</td>
</tr>
<tr>
<td>3-2</td>
<td>Evaluation of the conduction Nusselt number, $\text{Nu}_c$, showing the results of various methods and the correlatlon equation (3-6) used.</td>
</tr>
<tr>
<td>3-3</td>
<td>Experimental measurements of $\text{Nu}$ vs Ra cos $\theta$ for configuration A and configuration B.</td>
</tr>
<tr>
<td>3-4</td>
<td>Measurements of Nusselt number as a function of Ra cos $\theta$ for $A = 1$, showing results for $H = .5$ inch and for $H = 1.25$ inch.</td>
</tr>
<tr>
<td>3-5</td>
<td>Measurements of Nusselt number as a function of Ra cos $\theta$ for $A = 2.5$.</td>
</tr>
<tr>
<td>3-6</td>
<td>Measurements of Nusselt number as a function of Ra cos $\theta$ for $A = 4$.</td>
</tr>
<tr>
<td>3-7</td>
<td>Dimensional plot showing the heat transfer coefficient across an air layer having a plane upper surface and a V-corrugated lower surface, for various $L$ and $H$; $\theta = 45^\circ$, $T_h = 160$ F and $T_c = 60$ F.</td>
</tr>
<tr>
<td>4-1</td>
<td>Sketch showing pertinent dimensions of a rectangular-celled honeycomb.</td>
</tr>
<tr>
<td>4-2</td>
<td>Nusselt number versus Rayleigh number plots for a set of horizontal slits for experimental set-up number 1. (See Table 4-1 for full details on this set-up.)</td>
</tr>
<tr>
<td>4-3</td>
<td>Nusselt number versus Rayleigh number plots for experimental set-up number 2. (See Table 4-1 for full details of this set-up.)</td>
</tr>
<tr>
<td>4-4</td>
<td>Nusselt number versus Rayleigh number plots for experimental set-up number 3. (See Table 4-1 for full details of this set-up.)</td>
</tr>
<tr>
<td>4-5</td>
<td>Nusselt number versus Rayleigh number plots for experimental set-up number 4. (See Table 4-1 for full details of this set-up.)</td>
</tr>
</tbody>
</table>
4-6 Nusselt number versus Rayleigh number plots for experimental set-up number 5. (See Table 4-1 for full details of this set-up.)

4-7 Nusselt number versus Rayleigh number plots for experimental set-up number 6. (See Table 4-1 for full details of this set-up.)

4-8 Plot showing comparison of convection suppression of square-celled and horizontal slit honeycombs for same $A_E$.

4-9 Plot showing comparison of convection suppression of square-celled and horizontal slit honeycomb for same amount of honeycomb material.

4-10 Comparison of data at $\theta = 90^\circ$ with correlation equation 4-7. Values of $\gamma$ and $\eta$ used in this equation are given in Table 4.3. (See Table 4-1 for full details of each experimental set-up.)

LIST OF TABLES

Table 4-1 Description of the Rectangular Honeycomb and Plate Emissivities Investigated 39

Table 4-2 Measured and Predicted Critical Rayleigh Numbers for $\theta = 0^\circ$ 53

Table 4-3 Values of $\gamma$ for the Different Experimental Set Ups 58

Table 4-4 Permissible Values of $L$ and $L_{NS}$ for Suppression of Convection for Set Conditions $\text{NS}$ 62

Table 4-5 Table of Values of $F$ Found in Present Experiments 64

Table 4-6 Calculated Overall Heat Loss Coefficients for Honeycomb and Non-Honeycomb Situations 67
1. SUMMARY

This report describes the results of two studies, both of which are related to methods for improving the efficiency of flat plate solar collectors.

The first study deals with the free convective heat loss from a V-corrugated absorber plate to a plane transparent (glass) cover. Measurements carried out in the University of Waterloo Natural Convection Apparatus give Nusselt numbers as a function of Rayleigh number for angles of inclination of the plate from the horizontal of 0, 30, 45 and 60 degrees. The values of the depth ratio (the ratio of the average depth of the air layer to the depth of the V's) investigated were 1, 2.5 and 4; the angle of opening of the V's was 60°. It is concluded that the heat transfer is greater than that for two parallel plane plates having the same average spacing, by up to 50% for a depth ratio of unity. However, for the larger values of the depth ratio, the increase in heat transfer is slight. Correlation equations are given for the free convective heat transfer.

The second study deals with the free convective heat transfer occurring in a honeycomb solar collector in which the honeycomb consists simply of a set of horizontal partitions (or slits) oriented normal to the plate. Measurements were carried out on this problem on the University of Waterloo Natural Convection Apparatus. Aspect ratios (plate spacing to distance between partitions) of 3, 5 and 10 have been investigated, as have slit materials both opaque and transparent to thermal radiation. A significant conclusion is that for angles of inclination of the plate from the horizon-
tal ranging from 70 to 90 degrees, the horizontal slit honeycomb gives superior convection suppression to a square-celled honeycomb, for the same amount of honeycomb wall material. However, for angles less than 45°, horizontal slits are substantially less effective in convection suppression. Correlation equations are presented for the free convective heat transfer for angles of inclination ranging from 0° to 90°. Also presented are equations describing the condition for convection suppression by horizontal slits.
2. INTRODUCTION

Methods must be found for reducing heat losses from the conventional flat plate solar collector, if it is to perform more efficiently in solar heating and cooling systems for buildings. In particular, some additional mechanism or mechanisms must be incorporated into its design in order to substantially decrease losses from the collector top. These mechanisms must be economic, durable and practical. More important, they must not substantially decrease the solar radiation input to the collector plate.

The development of new such mechanisms, and the study of critical problems associated with mechanisms already suggested, is the broad objective of the research described in this report.

This broad objective is being pursued through two parallel studies:

(i) a study of free convection heat loss across the inclined air layer which exists in a flat plate solar collector having a V-corrugated absorber plate.

(ii) a study of free convective heat loss across the inclined air layer which exists in a flat plate solar collector incorporating a honeycomb of large plan aspect ratio.
The background and specific objectives of these individual studies are as follows:

**Study 1: Natural Convection across an Inclined Air Layer Confined Between one Flat Plate and one V-corrugated plate.**

A study on the free convective heat loss across an inclined air layer confined between two flat parallel plates was carried out at the University of Waterloo in the Phase I study [1]* and the resulting correlation equations [2,3] are now being used by many workers for characterizing the heat loss across the air gaps in flat plate solar collectors. The question now arises: is the equation still valid in the case where one or both of the bounding surfaces is not flat, but contains various protrusions (such as tubes), or is in some way roughened or corrugated?

As a first step in answering this question, the combination of the V-corrugated plate and the flat plate is being studied. Such a combination is of special interest since it occurs in the design of an efficient solar air heater developed in Australia [4,5]. In this air heater the absorber plate is V-corrugated so as to give two beneficial effects. First, the V-corrugating of the absorber plate improves its directional selectivity - particularly if the plate is slightly spectrally selective at the start [6]. Second, it exposes more surface area to the flowing air behind the plate, thereby enhancing the forced convective heat exchange between the circulating air and the absorber plate.

*Numbers in square brackets denote reference listed in Section 6.*
If the absorber plate is selective, the dominant heat loss is by free convection and hence the proper evaluation of this air heater requires an accurate knowledge of the free convective loss in this geometry.

(The combination of a V-corrugated plate and a flat plate is also of special interest in light of Phase 1 recent studies [1,7] indicating that V-corrugating the inner transparent cover of a two-cover collector will increase that cover's solar transmittance. In this instance it is the upper plate rather than the lower one which is V-corrugated.)

The objective of this study, therefore, is to generate a correlation equation for the free convective loss across this type of air layer, covering the range of variables applicable to solar collectors. A secondary objective is to compare this correlation equation with that applying to an air layer bounded by two plane surfaces, so as to quantitatively indicate the effect of protrusion and other forms of large-scale roughening of the absorber plate on the free convective loss.

Study 2: Natural Convection Across an Inclined Air Layer Constrained by a Honeycomb of Large Plan Aspect Ratio.

A number of recent studies [8,9] have demonstrated the improvement in solar collector efficiency obtained by incorporating a transparent honeycomb between the absorber plate and glass cover in a conventional one glass cover flat plate collector. As was indicated in an early analytical study [10], the improvement is due both to reduction in radiant
losses by the honeycomb, and due to suppression of the free convective loss by the honeycomb. In the Phase 1 study at the University of Waterloo [2,11], the suppression of the free convection loss by a square-celled honeycomb was studied in detail, and a correlation equation for the free convective loss covering the range of parameters suitable for solar collectors was obtained. The purpose of the present study is to extend the phase 1 study so as to include rectangular honeycombs with the long sides in the horizontal direction, the limiting case of which is a set of horizontal partitions, or slits. Included as part of this study is the effect of plate emissivity and honeycomb wall emissivity on the free convection.

The following sections of this report give the results obtained on these two studies; Study 1 is contained in Section 3, Study 2 is contained in Section 4. Full details of Study 1 are given in Reference [33]; and of Study 2 in Reference [34].
3. FREE CONVECTION ACROSS
INCLINED AIR LAYERS BOUNDED BY ONE
FLAT SURFACE AND ONE V-CORRUGATED SURFACE

3.1 Description of Problem

Figure 3-1 shows a sketch of the problem considered. Both configuration 'A' (upper sketch) and 'B' are considered and they will be denoted by 'A' and 'B' respectively. Note that in both cases heating is from below - i.e. the lower surface is the hot one. Configuration 'A' is that applying in a flat plate solar collector having a V-corrugated absorber plate. Configuration 'B' may be of interest in an instance where the transparent cover is V-corrugated. The air layer is assumed to be very extensive in the direction normal to the plane of the drawing. The angle of opening of the V's chosen for study was 60° since Australian air heater design studies have indicated this angle to be near optimal when one takes into account radiant absorption and emission by the plate, forced convection behind the plate and material and manufacturing considerations. The lower surface is maintained at a uniform temperature $T_h$, greater than that similarly maintained on the upper surface, $T_c$, by an amount $\Delta T = T_h - T_c$. The width of the layer, $W$ is assumed to be much greater than the spacing $L$. The ratio of $L$ to the depth of the V's, $H$, is termed the depth ratio, $A: A=L/H$. Three depth ratios were examined: $A=1, 2.5$ and $4$. Measurements were also taken at four angles of tilt with respect to the horizontal: $\theta=0, 30, 45$ and 60 degrees.

As far as heat transfer across the air layer is concerned, configurations 'A' and 'B' (Figure 3-1) are identical. That is, for the same sets
Sketch of the inclined air layers bounded by a V-corrugated plate and a flat plate. In configuration 'A' (upper sketch) the lower plate is V-corrugated and the upper is flat while in configuration 'B' (lower sketch) the reverse is the case. Both configurations were studied in the present study.
of parameters, virtually the same heat transfer will flow across the air layer in both instances. This statement, which we shall call, for brevity, reciprocity, was proven both theoretically and experimentally over the course of the study, as will be discussed shortly. The arrangement 'B' was more easily incorporated into the University of Waterloo Natural Convection Apparatus and hence most experiments were performed with this arrangement.

3.2 Previous Studies

A full review of the previous studies relevant to an inclined air layer bounded by two parallel flat plates is given in [2,3]. For the case where one of the plates is V-corrugated, the only relevant study to our knowledge is that of Chinnappa [12]. In this study, the angle of opening of the V's was 60 degrees, and heating was from below, with the V-corrugated plate being the lower plate. The angle of inclination of the flat plate from horizontal was zero degree. Since the air pressure was not varied (as is the case in the University of Waterloo Natural Convection Apparatus) only a narrow range in Rayleigh number was covered for any particular set-up. No effect of spacing ratio $A$, was observed, and heat transfer results were found to be intermediate between those for an air layer bounded between two parallel flat surface inclined at 0 degree and 60 degree from the horizontal.
3.3 Description of Experimental Apparatus

The University of Waterloo Natural Convection Apparatus which was used in this experiment is fully described elsewhere [2,11,13,14,15]. Its test section consists of two parallel flat copper plates one of which contains provision for measuring heat flux by means of a heat flux meter, and an electrically heated plate, called the "heater plate". In the present study, one of the flat plates (the one which does not incorporate the heat flux measurement capability) was replaced by a V-corrugated copper plate. This V-corrugated plate was constructed by folding a .032 inch thick copper sheet into the prescribed form and soldering tubes to its rear (top-side) in alternate V-grooves. These tubes were attached to headers at each end, and in the experiments, circulating water from a constant temperature bath is circulated through these tubes. Energy balance and fin calculations indicated that the plate would remain essentially isothermal (to within about .1F) at the heat flux rates experienced in the apparatus. Copper-constantan thermocouples were attached to representative points in the central area of the plate so as to determine the temperature difference between upper and lower plates, in the usual way for the apparatus. Comparisons of the readings of the various thermocouples verified the near-isothermality of the V-corrugated plate. The peripheral dimensions of the V-corrugated plate was the same as that of the lower plate - namely 22 inch by 24 inch. This gives a value for W, (Figure 3-1) of 24 inch. The height of the corrugations, H, was made .5 inch; a second V-corrugated plate was also made having an H = 1.5 inch and a sheet thickness of .064 inch.
The air layer was sealed around the periphery of the two plates by stretching aluminum foil between the plates and giving it a good thermal contact at each plate. This foil is of sufficient thickness (.005 inch) to ensure a linear temperature rise from one plate to the other, thereby ensuring no net loss of heat from the air layer to the surrounds.

Most experimental runs were made with configuration 'B', using the guarded heater plate imbedded in the flat lower plate to measure the heat flux. Some measurements were also made on configuration 'A', in which case the plate containing the guarded heater plate was used as the upper plate. In this situation, cooling rather than heating would have to be applied to the heater plate to keep it at the same temperature as the rest of the upper plate. Since such cooling was not possible, the heater plate could deviate from the upper plate by up to 1.5°F (or 10% of the total temperature difference) for these runs, depending on the Nusselt number. Since the deviation represents an experimental uncertainty, experiments using the situation shown in Figure 1B were preferred.

3.4 Dimensional Analysis

For the present set of experiments the Nusselt number is a function of the following dimensionless groups:

\[ Nu = Nu (Ra, A, \theta) \]  

(3-1)
where $Ra$ is the Rayleigh number defined by:

$$Ra = \frac{g \beta AT L^3}{\nu \lambda}$$  \hspace{1cm} (3-2)

where $g$ = acceleration of gravity and $\beta$ the thermal expansion coefficient, $\nu$ the kinematic viscosity and $\lambda$ the thermal diffusivity of air. The depth ratio $A = L/H$. The angle of inclination $\theta$ is as defined in Figure 3-1. The Nusselt number is defined by:

$$Nu = \frac{qL}{k \Delta T}$$  \hspace{1cm} (3-3)

where $q$ is the heat flux across the air layer (based on unit area of flat surface on the flat plate), exclusive of radiative exchange, and $k$ is the thermal conductivity of air. Fluid properties are evaluated at the arithmetic mean of the two plate temperatures.

Other dimensionless groups of interest are: the Prandtl number, $Pr$, which is fixed at that for air at moderate temperatures, $Pr \approx 0.71$, and the angle of opening of the V's, which is fixed at 60 degree.

The aspect ratio $W/L$ is assumed to be sufficiently large in the experiments that the effects of the periphery are not felt at the central area where the heat flux is measured. Thus the results are expected to be applicable to that situation existing in the limit of large $W/L$. Our experience gained on studies having two flat surfaces [2,14] have shown that this expectation is satisfied provided $\theta \leq 60$ degree.

† Conventional flat plate solar collectors can be assumed to fall near this limit.
For the range $60 < \theta \leq 90$ degree the effects of the periphery are felt in the central area. Consequently measurements were not taken for $\theta > 60$ degrees.

3.5 Range of Measurements

The approximate range of variables of interest for the design of solar collectors is as follows:

$$10^3 < Ra < 10^6$$

$$1 \leq A \leq 4$$

$$0 \leq \theta \leq 90 \text{ degree}$$

Except for the range $60 < \theta \leq 90$ alluded to above, this range was attempted to be covered in the present experiments. Measurements were taken at values for $\theta$ of 0, 30, 45 and 60 degree, and for $A$ of 1.0, 2.5 and 4.

As is customary on the University of Waterloo Apparatus, the Rayleigh number was varied by varying the air pressure, keeping $L$ and $\Delta T$ fixed. This permits the full variation of the Rayleigh number while minimizing the number of experimental set-ups necessary and also minimizing errors associated with stray effects. The full range in Rayleigh number could be covered with the V-corrugated plate having $H = 0.5$ inch except, in the case $A = 1$, for the range $10^5 < Ra < 10^6$. The V-corrugated plate having $H = 1.5$ inch permitted this range to be covered.
3.6 Evaluation of the Nusselt Number at Low Rayleigh Number -
the Conduction Limit

At very low Ra, the free convection in the air layer is virtually eliminated and heat transfer across the air layer is by conduction and radiation only. Since it is difficult to separate out these two components (radiation and conduction) in the heat flux measurements, it was found preferable to determine the purely conductive heat transfer by other means - namely by analysis and by electrical analog experiments. The purely conductive heat transfer $q_c$ we characterize by the Nusselt number $\text{Nu}_c = q_c L/k\Delta T$. Upper and lower bounds for $\text{Nu}_c$ were established by the method of [16] and of Elrod [17], yielding:

$$\text{A} \ln \left( \frac{2A+1}{2A-1} \right) < \text{Nu}_c < \frac{3\sqrt{3}}{\pi} \text{A} \ln \left( \frac{2A+1}{2A-1} \right),$$

(3-4)

so that:

$$\text{Nu}_c = 1.246 \text{A} \ln \left( \frac{2A+1}{2A-1} \right) \pm 25\%$$

(3-5)

A plot of 3-4 is shown in Figure 3.2. To establish more precisely the conduction transfer, $\text{Nu}_c$, an electric analog experiment was conducted using electrically conducting paper [18]. These results, converted to Nusselt number, are plotted in Fig. 3-2. In addition an analytical solution was attempted using conformal mapping and the Schwarz-Christoffel transformation. The resulting points are also plotted in Figure 3-2. On the basis of all this information an equation fitting $\text{Nu}_c$ to $A$ was determined:
Evaluation of the conduction Nusselt number, \( \text{Nu}_c \), showing the results of various methods and the correlation equation (3-6) used.
\[ \text{Nu}_c = A \ln \left( \frac{2A + 1}{2A - 1} \right) / \left[ 1 - \frac{3025}{A} + \frac{683}{A^2} \right] \]  

(3-6)

The guidance for the form of this equation comes from equation (3-5) and the fact that \( \text{Nu}_c \to 1 \) as \( A \to \infty \). A comparison between the various data points and this equation is made in Fig. 3-2. It will be noted that as \( A \) approaches 0.5, \( \text{Nu}_c \) becomes infinitely large because the bottom of the corrugations come into contact with the flat plate. Equation (3-6) should be valid to within 5% in \( 1 \leq A \leq 4 \).

3.7 **Evaluation of Radiative Heat Transfer Correction**

With the plates in the horizontal position (\( \theta = 0 \)) and at the lowest Rayleigh number possible (\( Ra \approx 10 \)) the heat transfer across the air gap was measured. Since this is substantially below the observed critical Rayleigh number, it is assumed that the heat flow is by radiation and conduction alone. Subtracting the conduction component, using the results of the previous section, yields the heat transfer coefficient for radiation. Since the plate temperatures remained essentially unchanged over the entire \( Ra \) and \( \theta \) range, the heat transfer by radiation can be calculated using this heat transfer coefficient for all subsequent measurements, including those where free convection is present. Therefore, the radiative transfer could be subtracted away from the total heat transfer to yield the desired Nusselt number for convection plus conduction, \( \text{Nu} \).
3.8 Experimental Results

Figure 3-3 shows the experimental results for both cases, A and B (Figure 3-1), for spacing ratio $A = 4$ and $\theta = 45^\circ$. Nearly identical results are seen to have been obtained for both cases for the same Rayleigh number. This plot gives an experimental proof that reciprocity is valid within experimental uncertainty. The above experiment was repeated with $\theta = 0^\circ$. In this case, some significant differences were observed (15%) between cases A and R. but these were shown to be due to the non-isothermality of the upper plate in case A. Full details are given in [33]. All further experiments were performed using the configuration shown in Figure 1B.

The experimental measurements of Nusselt number, $Nu$, and Rayleigh number, $Ra$, are plotted in Figures 3-4, 3-5 and 3-6, for spacing ratios, $A$, of 1, 2.5 and 4, respectively. The plots are given in the form of $Nu$ vs. $Ra \cos \theta$.

For $A = 1$, the data points from the V-corrugated plate with small corrugations, agree with those from the second plate with larger corrugations. This substantiates that the Nusselt number is only dependent on the aspect ratio, $A$, (and not separately dependent on $H$), as indicated by the dimensional analysis. For $\theta = 0$, the Nusselt number was constant and equal to $Nu_c$, indicating absence of convection up to a critical Rayleigh number, $Ra_c$, after which convection started and a sudden increase in $Nu$ was observed with the increase in $Ra$. For $\theta > 0^\circ$, a gradual increase in Nusselt number was observed from the start and $Nu$ increased gradually with $Ra$, indicating perhaps an effect of the base.

†A theoretical proof is given in [33].
Figure 3-3

Experimental measurements of Nu vs Ra cos θ for configuration A and configuration B.
Measurements of Nusselt number as a function of $\text{Ra} \cos \theta$
for $A = 1$, showing results for $H = .5$ inch and for $H = 1.25$ inch.
Measurements of Nusselt number as a function of Ra cos θ for A = 2.5.
Figure 3-6

Measurements of Nusselt number as a function of Ra cos θ for A = 4.
flow which is present for any finite value of Rayleigh number. This is observed only for \( A = 1 \).

3.9 Discussion of Results

It is of interest to observe how well the correlation equation for \( \text{Nu} \) developed previously [2], based on two flat plates compares with the present measurements for the same spacing. The dashed lines in Figs. 3-4 and 3-6 are plotted from this equation. It is seen from the Figures that the conduction Nusselt number is higher by 24\%, 7\% and 4\% for \( A = 1, 2.5, \) and 4.0 respectively. The Rayleigh numbers at which convection begins (for \( \theta = 0^\circ \)) or starts to play a significant role (for \( \theta > 0^\circ \)) are slightly higher in the V-corrugated case, indicating a certain amount of convection suppression by the V's. There is reasonable agreement between the two cases over the region \( 3 \times 10^3 \leq \text{Ra} \leq 3 \times 10^4 \) for \( A = 2.5 \) and 4 and in a very narrow band about \( \text{Ra} = 8 \times 10^3 \) for \( A = 1.0 \). At still higher \( \text{Ra} \), the data fall progressively higher than the corresponding flat-plate data. It is speculated that this is caused by cold jets of air departing from the tips between the V-grooves and penetrating the "boundary layer" on the opposite plate. If this is the mechanism, then it would be expected that the Nusselt numbers at much higher Rayleigh numbers would not continue to depart from the solid lines, but rather approach from above asymptotes parallel to those for the parallel flat plates.

3.10 Correlation Equation

The three variable function: \( \text{Nu} = \text{Nu} (\text{Ra}, \theta, A) \) is obviously
a very complex one and it was not found possible to express it in any simple way over the full experimental range of parameters. However, a single equation was found which covers most of the range of interest. It is:

$$\text{Nu} = \text{Nu}_c + K[1 - \frac{\text{Ra}_c}{\text{Ra} \cos \theta}]^* (1 - \frac{\text{Ra}_c (\sin 1.8\theta)^{1.6}}{\text{Ra} \cos \theta})$$

$$+ B \left[ \left( \frac{\text{Ra}_c \cos \theta}{\text{Ra}_t} \right)^{1/3} - 1 \right]^*$$

where $\text{Nu}_c$ is given by equation (3-6) and:

$$\text{Ra}_c = 1708 \left[ 1 + \frac{.0360}{A} + \frac{2.69}{A^2} - \frac{1.703}{A^3} \right]$$

(3-8)

$$K = \frac{2459}{\text{Ra}_c} \left[ 1 - \frac{.195}{A} + \frac{5.97}{A^2} - \frac{4.16}{A^3} \right]$$

(3-9)

$$B = 2.225 - .01226 \theta + .340 \times 10^{-3} \theta^2$$

(3-10)

$$\text{Ra}_t = 11,290 \left[ 1 + .204 \sin [4.50 (\theta - 37.8^\circ)] \right]$$

(3-11)

In the equations $\theta$ must be expressed in degrees. The square brackets superscripted by a solid circle, like so: $[\ ]^*$ denote that if the argument inside the bracket is negative, the quantity is to be taken as zero, i.e.:

$$[X]^* = \frac{|X| + X}{2}$$

where $X$ is any quantity.
The comparison of the data with equation (3-7) is shown in Figures (3-4) (3-5) and (3-6). Agreement is seen to be good (within 8%) except for the narrow range $10^3 < Ra < 10^4$ for $A = 1$. In this region errors of up to 30% are observed.

The first term on the right hand side of equation (3-7), namely $Nu_c$, is the purely conductive (i.e. non-convective) heat transfer across the air gap. Provided $Ra < Ra_c$ it is the only term to be considered. At $Ra \geq Ra_c$ convective motion starts to take place and the corresponding rise in $Nu$, is given by the second term. The concept of a critical Rayleigh number governing the start of convective motion - one that has been fully developed in many studies for plane air layers - appears to be equally relevant to V-corrugated air-layers. In the present study, the value of $Ra_c$ was found experimentally to be a function of spacing ratio, $A$, and equation (3-8) is an empirical fit of the measured $Ra_c$'s (for $\theta = 0$) for the 3 values of $A$ tested. The limit $A + \infty$ corresponds to two flat plates, and equation (3-8) gives the accepted value of 1708 for this case. The rate of rise of $Nu$ with $Ra$ once convection has just set in is governed by the parameter $K$. For a plane air layer, $K$ is known to be 1.44. In the present study $K$ was found to be a monotonically decreasing function of the spacing ratio, $A$. Equation (3-9) is an empirical fit of the measured values of $K$ at $\theta = 0$, to $A$, chosen so as to ensure $K + 1.44$ as $A + \infty$. (Methods of obtaining measured values of $K$ and $Ra_c$ from the data points are discussed in [14], and [33].) The form of the second term in equation (3-7) is identical to that used earlier for plane air layers [2].
Figure 3-7

Dimensional plot showing the heat transfer coefficient across an air layer having a plane upper surface and a V-corrugated lower surface, for various L and H; $\theta = 45^\circ$, $T_h = 160^\circ F$ and $T_c = 60^\circ F$.

CONDITIONS: $T_c = 15.5^\circ C (60^\circ F)$
$T_h = 71^\circ C (160^\circ F)$
$\theta = 45^\circ$
The form of the third term in (3-7) was also stimulated by the form of the equation for plane air layers [2]. It should represent the limit of fully-turbulent heat transfer where turbulent boundary layers exist on all bounding surfaces to the air layer and a fully-mixed turbulent core exists in the central region. The quantity $Ra_t/\cos \theta$ represents the criterion on $Ra$ above which this term first contributes to $Nu$. For $Ra < Ra_t/\cos \theta$ this term does not contribute. For plane air layers the value of $Ra_t$ was found previously to be a constant (5830), independent of $\theta$. However for V-corrugated air layers a significant dependence on $\theta$ was noted, although only slight dependence on A was noted. Values of $Ra_t$ were determined for each angle by minimizing the root mean square deviation of equation (3-7) from the measured data points. The resulting values of $Ra_t$ were then fitted to $\theta$ by an empirical fit and the result is equation (3-11). The quantity B would be unity, if the analogy with the plane air layer equation were exact. However, attempts to fit the equation to the data with $B = 1$ resulted in such large deviations that an introduction of non-unity values was necessitated. Values of B were arrived at in the same way as those of $Ra_t$ - i.e. by minimizing the root mean square departure of equation (3-7) from the data. The resulting values of B were then fitted to $\theta$, yielding equation (3-10).

Attempts were made to estimate the expected value of $Ra_t$ and B from the methods outlined in [19]. The predicted value using this method was $Ra_t = 3377$ and $B = 1$. The reason for the departure of these values from those found from the experiment are not known with confidence, but are expected to be due to the fact that the Rayleigh number in the ex-
periments was not sufficiently large to approach the fully turbulent regime. In the case of the plane air layer at moderate $R_a$, jets of fluid are known to leave one boundary layer and penetrate the opposite boundary layer. The existence of the fully turbulent regime cannot be established until the spacing is sufficiently large (the Rayleigh number is sufficiently large) that these jets cannot penetrate the full width of the layer. It is felt that the $V$'s promote strong jets so that the Rayleigh number sufficient to establish the fully-turbulent regime is increased over that for plane layers.

3.11 Application to the Design of Solar Collectors

Although a $V$-corrugated absorber plate has improved radiative properties over that for a flat plate, and although it has an increased area for forced convection at its rear (when used in solar air heaters), this study has shown that the free convective heat losses from such a plate will be generally higher than that for a flat absorber plate. However, by properly choosing the dimensions of $L$ and $H$, this increase in free convective heat loss can be minimized. In order to choose $L$ and $H$ it is recommended that the designer first prepare a graph such as is shown in Figure 3-7. Based on equation (3-7) this plot shows the free convective heat transfer coefficient, $h$, as a function of $L$ and $H$ for the conditions: $\theta = 45^\circ$, $T_c = 60F (15.5C)$ and $T_h = 160F (71^\circC)$. Also shown is the free-convective heat transfer coefficient for two parallel flat plates, as given by reference [2]. The region $L = 1$ to $1.5$ cm for $H = 1$ cm was taken from Figure 3-4 rather than equation 3-7 since this
corresponds to the region of poor agreement to that equation. The restriction on equation 3-7 that \( A \leq 1 \) prevents the lines for constant \( H \) from extending beyond the point where \( L = H \). However, equation (3-6) gives a lower bound on \( \text{Nu}_c \) and the corresponding lower bounds on \( h \) are sketched in the figure. With the assist of these lower bounds, interpolated curves have been sketched for \( .5H < L < H. \)

For the given conditions, it appears that a good design point from a purely free convective point of view is: \( L \approx 1.5 \text{ cm}, H \approx 1 \text{ cm.} \) This will produce a free convective loss coefficient of \( h \approx 4 \text{ W/m}^2 \text{ K}, \) which is about 25% higher than that which would be the case for a plane air layer having \( L = .8 \text{ cm}, \) namely \( h = 3.2 \text{ W/m}^2 \text{ K}. \) However, it should be noted that in the latter case, the average value of \( h \) over the year will not be as low as 3.2 W/m\(^2\) K since it is unlikely that \( T_h \) and \( T_c \) will be constant in practice.
4. FREE CONVECTION
ACROSS AIR LAYERS CONSTRAINED BY
A HONEYCOMB OF LARGE PLAN ASPECT RATIO

4.1 Problem Background and Description
4.1.1 Geometry

The method used to specify the various dimensions and aspect ratios will first be given. Referring to Figure 4-1, the honeycomb panel is assumed to be facing south. The dimension of the honeycomb cell in the east-west direction is $L_{EW}$, in the north-south direction $L_{NS}$, and the depth of the honeycomb panel, $L$. The angle of tilt of the panel from horizontal is $\theta$. The two important aspect ratios namely $A_E$ and $A_p$, are defined by:

$$A_E = \frac{L}{L_{NS}}$$

$$A_p = \frac{L_{EW}}{L_{NS}}$$

From a purely free convection point of view, rectangular honeycombs may not be desirable for a horizontal or near-horizontal (say up to $\theta = 15^o$) collector inclination, as may be used for summer cooling in the southern U.S. This is because there is an equal tendency for free convective cells to be formed in any direction - therefore a laterally symmetric cell shape such as a square or hexagonal cell appears to be called for.

For moderate collector tilts (range $\theta = 15^o$ to $45^o$), the first incipient fluid motion is rolls with axis in the north-south upslope
Figure 4-1
Sketch showing pertinent dimensions of a rectangular-celled honeycomb.
Therefore it would appear that for this inclination, honeycombs with $A_p < 1$ are called for. However, since there is still some tendency for rolls with axis in the east-west direction, $A_p$ cannot be made equal to zero. From the point of view of minimizing the quantity of honeycomb wall material per unit area (and hence minimizing the honeycomb cost, and maximizing its solar transmittance) it would appear that the optimum shape of cell would be a rectangle with $L_{EW}$ of sufficient size to "just" suppress the incipient north-south rolls, and $L_{NS}$ of sufficient size to "just" suppress the incipient east-west rolls. That is, there will be an "optimum" value of the plan aspect ratio, $A_p$, which will be called $A_{p\text{'opt}}$. Since the relative stability of the two rolls is dependent upon the angle of tilt, $\theta$, $A_{p\text{'opt}}$ is a function of $\theta$. (At $\theta = 0$, the tendency is the same for both directions; therefore $A_{p\text{'opt}}$ for this angle is unity). The exact nature of the function $A_{p\text{'opt}}(\theta)$ is not known. However, gaining of this knowledge is of some importance to the design of honeycomb solar collectors.

For angles of tilt near vertical ($70 < \theta < 90^\circ$), a reversal of the two rolls takes place. For these angles the flow situation is more unstable to east-west rolls than to north-south rolls. Therefore $A_{p\text{'opt}} > 1$. In fact at $\theta = 90^\circ$, there are theoretical grounds for believing that there is no tendency for north-south rolls so that $A_{p\text{'opt}} = \infty$. In this case the honeycomb becomes simply a set of horizontal slits - rather like a venetian blind.

This range of angle of inclination is of some importance since it corresponds to near-optimal for winter heating in northern United States
and Canada. For these inclinations, then, what may be needed is not strictly a honeycomb, but a set of horizontal slits.* Consequently this is the geometry being investigated in the present study.

4.1.2 Radiation Aspects

Also investigated is the effect of long-wave radiation on the suppression of free convection by honeycombs of the horizontal slit type. In particular the effect of the emissivity of the honeycomb wall material, ε, is investigated. Also investigated is the effect of the emissivity, denoted by ε₁ and ε₂, of each of the two plates which form the upper and lower bounds for the honeycomb.

In application the honeycomb is placed between the absorber plate and the glass cover in a flat plate solar collector. Thus the two flat surfaces bounding the honeycomb are the absorber plate and the glass cover. Provided the absorber plate is black paint and the glass cover is untreated, the emissivity of both of the bounding surfaces is approximately .9. If the absorber surface has a selective surface applied to it, then the emissivity of the lower bounding surface can be as low as .05 for a good selective surface, while that of the glass remains at .9: hence ε₁ = .05 to .15, ε₂ = .9. If in addition the glass cover is given a tin oxide coating to form a heat mirror, then the emissivity of the glass can be as low as .1 so that in this case ε₁ = .05 to .15, ε₂ = .1 to .2. Thus in practice a full range of plate emissivities can be expected.

*For the purposes of this report we will consider a set of horizontal slits to be a special case of a honeycomb.
The effect of these emissivities on the free convection will therefore be investigated.

4.2 Previous Studies

Catton and co-workers [21,22], have performed analytical and experimental investigations of flow in rectangular slots similar to that enclosed by horizontal slits. However, a sufficient range of aspect ratios is not covered and the Prandtl number is not that of interest. Some work has been carried out by Probert and Ward [23] on what is in fact the effect of horizontal slits in a vertical air gap. Positive results (i.e. suppression of free convective heat transfer) was found but the results are not in sufficient detail to be used for design.

J. E. Hart [24] has examined in some detail a geometry which, although treated as a problem in geophysics, is very close to a single cell in a set of horizontal slits with \( \theta = 90^\circ \). A number of workers, e.g., references [25,26,27] have examined a single cell with \( \theta = 90^\circ \) by numerical modelling and experiments; however, the side-wall boundary conditions are not those applying in solar collector. Charters and Koutsoheras [28] report interesting numerical modelling of the problem covering all \( \theta \) but have not carried out experiments. In summary, the problem has been examined but there is not

** The previous studies noted in this section will not include the relevant literature corresponding to the problem where \( \theta = 0 \). This problem is reviewed in [32].
sufficient information to design with confidence, nor to seek optimized
designs. Good experimental data are particularly lacking.

Recently Arnold et al [29] reported a set of experiments on
rectangular honeycombs having approximate values for $A_p$ of 2, 4 and 8 and
for $A_E$ of 4. The fluid was an opaque liquid and the wall conductivity
was low. The radiant coupling, present in honeycombs with air, was not
accounted for and consequently critical Ra's were much lower than would
be expected in solar collectors for the same aspect ratio. An important
conclusion was that provided $A_p$ is greater or equal to 4, it has no effect
on the heat transfer. This conclusion may be limited to the value of $A_E$
investigated, as well as to the values of the other pertinent groups.

The important effect of radiative exchange on the free convection
in a honeycomb cell was first pointed out by Sun and Edwards [30] and
studied in greater depth by Sun [31]. They showed that the radiation
damps the free convective motion by stabilizing the cellular flow. This
effect results from a coupling of radiative and convective exchange at the
cell side-wall. An elemental area on the side wall is in radiative heat
exchange with the top and bottom plates, and with other points on the
side-wall. It is also in conductive and free convective exchange with the gas.
Near the critical condition incipient cellular motion will tend to alter
the temperature along the side wall; the radiation exchange will tend to
resist this alteration, thereby stabilizing the flow and producing higher
critical Rayleigh numbers.

The work on radiative interaction done by Edwards and Sun is
limited to the horizontal condition, $\theta=0$. Very little is known about the
radiative effect at other angles, particularly near $\theta = 90^\circ$. Moreover
their study did not investigate the effect of boundary plate emissivity,
$\epsilon_p^1$ and $\epsilon_p^2$.

4.3 Description of Apparatus

The University of Waterloo Natural Convective Apparatus [13, 14] was used to measure free convective heat transfer across horizontal slit honeycomb. The set of horizontal slits was constructed by winding polyethylene film tightly on a frame which was attached to the periphery of the copper plates (details are given in [34]). The fluid contained between the slits was air. The value of $L_{EW}$ (Figure 4-1) was fixed at 22 inch since this is the width of the copper plates. The value of $L_{NS}$ was made 1/2 inch so that $A_p = 44$. Two types of polyethylene film were used. The first was transparent polyethylene having a thickness of .0015 inch and an emissivity estimated at .13 [11]; the second was black opaque polyethylene having a thickness of .006 inch and an emissivity of .9, as measured on a Gier Dunkle DB100 reflectometer.

The polished copper plates normally used on the apparatus as upper and lower bounding surfaces for the honeycomb, had emissivities of .065. A method was developed for altering the emissivity of these plates by first applying a strippable lacquer, and then applying a paint on top of the lacquer. (The lacquer can be easily stripped from the plates
to return the polished copper plates when desired). A coating of Nexel velvet black paint was used in this study, giving the plates an emissivity of .9 for certain tests.

4.4 Dimensional Analysis and Range of Measurements

The free convective heat transfer across the honeycomb is characterized by the Nusselt number, Nu, defined by:

$$\text{Nu} = \frac{hL}{k_f}$$

(See Nomenclature for full definition of all terms)

Briefly Nu represents the free convective heat transfer across the honeycomb compared to that which would exist if the air were stagnant. Dimensional analysis shows Nu to be a function of the following dimensionless groups:

$$\text{Nu} = \text{Nu} (Ra, Pr, A_E, A_p, \theta, C, H, N, \varepsilon, \varepsilon_{p1}, \varepsilon_{p2})$$

The quantities $A_E$, $A_p$, $\theta$, $\varepsilon$, $\varepsilon_{p1}$, $\varepsilon_{p2}$ have been discussed and defined previously. $Ra$ represents the Rayleigh number:

$$Ra = \frac{g \beta (T_h - T_c) L^3}{\nu \alpha}$$
Pr represents the Prandtl number, \( Pr = \nu/\alpha \) which is .71 for air at moderate temperatures. The group C and H represent the effect of heat conduction in the honeycomb wall on the free convection. C represents heat conduction along the wall and is defined by:

\[
C = \frac{k_f L}{k_w \delta}
\]

H represents heat conduction across the wall (between adjacent cells) and is defined by:

\[
H = \frac{k_f \delta}{k_w L}
\]

The group N (together with the emissivities \( \epsilon, \epsilon_{p1} \) and \( \epsilon_{p2} \)) represents the strength of the effect of the radiant transfer on the convective transfer:

\[
N = \frac{4 \sigma T_m^3 L}{k_f}
\]

where \( T_m = (T_h + T_c)/2 \) and \( \sigma \) is the Stefan-Boltzman constant. (If \( T_h/T_c \) is not approximately unity then it also should be included in the list. All surfaces are assumed to be radiantly grey).
As far as possible the range of values of each of the parameters examined in this study are chosen to cover those of interest in the design of honeycomb solar collectors. Table 4.1 summarizes the descriptions of the six combinations of honeycombs and bounding plate emissivities tested. For each of the six experimental setups noted in that Table, a range of Ra and θ was covered.

A range of $A_E$ of 3 to 10 was covered. Honeycombs having $A_E < 3$ generally give such a small gain to performance of solar collectors that they are not justified. Honeycombs having $A_E$ greater than 10 incorporate so much honeycomb material as to be not cost-effective. The value of $A_p$ was fixed at 44 but according to the results of the study by Arnold et al [29], they should be valid for any $A_p$ greater or equal to 4, i.e. for $A_p \geq 4$. The effect of $C$ is expected to be relatively minor, provided it is large ($C > -10$), since radiation rather than conduction should by the dominant non-convective heat transfer mechanism by the wall, provided the walls are thin and made of a poor heat conductor as is required of a honeycomb in a solar collector. Similarly the effect of $H$ should be small provided it is small, as it was in the experiments (Table 4-1) and as it should be in properly designed solar honeycombs. The range of values of $\epsilon, \epsilon_p1$ and $\epsilon_p2$ covered in the experiments should include many combinations of interest in solar collectors.

Moreover, for $A_E = 1$ or 2, the honeycomb will augment the free convective heat transfer if $\theta > \sim 30^\circ$. 
### TABLE 4-1
**DESCRIPTION OF THE RECTANGULAR HONEYCOMB AND PLATE EMISSIVITIES INVESTIGATED**

<table>
<thead>
<tr>
<th>Experimental Set up No.</th>
<th>$2d$ inch</th>
<th>$L$ inch</th>
<th>$L_{NS}$ inch</th>
<th>$A_E$</th>
<th>$\varepsilon$</th>
<th>$\varepsilon_{p1}$</th>
<th>$\varepsilon_{p2}$</th>
<th>$C$</th>
<th>$N$</th>
<th>$H$ X10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.0015</td>
<td>2.5</td>
<td>.5</td>
<td>5</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>166</td>
<td>15.2</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>.0015</td>
<td>5</td>
<td>.5</td>
<td>10</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>332</td>
<td>30.4</td>
<td>.75</td>
</tr>
<tr>
<td>3</td>
<td>.0015</td>
<td>1.5</td>
<td>.5</td>
<td>3</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>100</td>
<td>9.1</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>.006</td>
<td>2.5</td>
<td>.5</td>
<td>5</td>
<td>.9</td>
<td>.065</td>
<td>.065</td>
<td>42</td>
<td>15.2</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>.006</td>
<td>2.5</td>
<td>.5</td>
<td>5</td>
<td>.9</td>
<td>.065</td>
<td>.065</td>
<td>42</td>
<td>15.2</td>
<td>6</td>
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<td>5</td>
<td>.9</td>
<td>.065</td>
<td>.065</td>
<td>42</td>
<td>15.2</td>
<td>6</td>
</tr>
</tbody>
</table>
For each experimental setup the Rayleigh number range covered was approximately: $5 \times 10^3 < Ra < 10^7$. For $Ra < 5 \times 10^3$ the air is virtually stagnant for the honeycombs examined and heat transfer through it is virtually conductive; consequently $Nu = 1$ and measurements having $Ra < 5 \times 10^3$ are unnecessary. The angle range covered was from 0 to 90°. Since horizontal slits are expected to be most valuable in collectors mounted nearly vertically, the primary angles tested were near 90°; they were: $\theta = 60°$, 75° and 90°. In addition for each setup a run was made at 0°, and in some cases, runs were made at 30°. As will be discussed later, a scaling law enabled the results at $\theta = 0°$ to be predictive of the range $0 \leq \theta \leq 60°$.

4.5 Results

The experimental results for experimental setups 1 to 6 are shown in Figures 4.2 to 4.7 respectively. They show plots of the measured Nusselt number ($Nu$) as a function of Rayleigh number $Ra$. The different angles are represented by different plotting symbols (see legend). The upper graph shows $Nu$ vs $Ra$ for $\theta = 90°$. The data for $\theta < 90°$ are plotted against $Ra \cos \theta$ rather than $Ra$, in the lower graph. By plotting the latter results in this way it is seen that the data for different angles are made to lie closely together. Also shown on each of the lower plots is the convective heat transfer which would exist if no honeycomb were present, as predicted by the correlation equation of [2]. Thus comparison of this curve with the data shows the degree of convection suppression by the honeycomb.
Figure 4-2

Nusselt number versus Rayleigh number plots for a set of horizontal slits for experimental set-up number 1. (See Table 4-1 for full details on this set-up.)
Figure 4-3

Nusselt number versus Rayleigh number plots for experimental set-up number 2. (See Table 4-1 for full details of this set-up.)
Figure 4-4

Nusselt number versus Rayleigh number plots for experimental set-up number 3. (See Table 4-1 for full details of this set-up.)
Figure 4-5

Nusselt number versus Rayleigh number plots for experimental set-up number 4. (See Table 4-1 for full details of this set-up.)
Figure 4-6

Nusselt number versus Rayleigh number plots for experimental set-up number 5. (See Table 4-1 for full details of this set-up.)
Nusselt number versus Rayleigh number plots for experimental set-up number 6. (See Table 4-1 for full details of this set-up.)
As is often found with honeycombs, for $\theta < 90^\circ$ the Nusselt number is effectively constant at unity until a critical value is reached at which free convection sets in and the Nusselt number rises sharply. The Rayleigh number at which this rise occurs is called the critical Rayleigh numbers, denoted by $Ra_c$. This sharp rise is identified with the "top-heavy" instability caused by warm, light fluid situated under cold, heavy fluid. This effect is governed by the component of gravity normal to the bounding hot and cold plates, $g \cos \theta$ and is maximum at $\theta = 0$. On the other hand, the curve at $\theta = 90^\circ$ shows a much more gradual rise in Nusselt number. In this case the convective heat transfer is identified with the "base flow" - a unicellular motion in which the fluid rises along the hot bounding plate and falls along the cold bounding plate. Driven by the component of gravity parallel to the bounding plates, $g \sin \theta$, this flow is always present provided $\theta > 0$. However by inspection of the experimental results, its effect is seen to be insignificant for $\theta < 75^\circ$, since the "top-heavy" instability dominates before its effect is felt.

Experimental setups 1, 2 and 3 are designed to show the effect of aspect ratio $A_E$ on the convection suppression. It can be seen that, as expected, the convection suppression, as exemplified, for example by $Ra_c$, increases with increasing values of $A_E$.

A comparison between the convection suppression by square-celled vis-a-vis rectangular-celled honeycombs is made possible by Figures 4.8 and 4.9. In Figure 4.8 the results for experimental setup no 1 ($A_E = 5$) are re-plotted vs $Ra$ with a different plot for each angle. Shown on the same graphs are the results for a square-celled honeycomb having the same
Figure 4-8

Plot showing comparison of convection suppression of square-celled and horizontal slit honeycombs for same $A_E$. 

• PRESENT EXPERIMENT, HORIZONTAL SLIT HONEYCOMB $A_E=5$, $A_p \approx \infty$

\[ \theta = 90^\circ \]

CORRELATION EQ. SQUARE CELLED HONEYCOMB $A_E=5$, $A_p=1$

\[ \theta = 75^\circ \]

\[ \theta = 60^\circ \]

\[ \theta = 0^\circ \]
Plot showing comparison of convection suppression of square-celled and horizontal slit honeycomb for same amount of honeycomb material.
value of aspect ratio $A_E$, namely $A_E = 5$, which is shown by a solid line. This line is based on the correlation equation developed in the phase 1 study, [1,11]. The values of $\varepsilon$, $\varepsilon_{p1}$ and $\varepsilon_{p2}$ are matched in both cases. The square-celled honeycomb is seen to give greater convection suppression at all angles but its advantage over a set of horizontal slits decreases as the angle of inclination, $\theta$, increases. It must be remembered that a square-celled honeycomb having $A_E = 5$ contains twice the amount of film material as a set of horizontal slits with $A_E = 5$. The two honeycomb geometries are compared for the same amount of film material in Figure 4-3. It is seen that for $\theta < 75^\circ$ the square cells still give better suppression but for $\theta > 75^\circ$ the horizontal slits give slightly greater suppression. At $\theta = 75^\circ$ the suppression is about the same in both cases.

The effect of honeycomb wall emissivity, $\varepsilon$, is shown by comparing Figures 4.2 and 4.5. For $\theta < 75^\circ$ the free convective heat transfer across the honeycomb is seen to have been reduced by a higher wall emissivity, the critical Rayleigh number being increased by a factor of 2.3 by increasing the wall emissivity from .13 to .9. However at $\theta = 90^\circ$, very little if any effect due to the change in $\varepsilon$ is observed.

The effects of plate emissivity, $\varepsilon_{p1}$, $\varepsilon_{p2}$ can be seen in Figures 4.5 to 4.7. A weak but significant effect of these variables on the convection is observed. Starting from the case where both $\varepsilon_{p1}$, and $\varepsilon_{p2}$ are .9, Fig. 4.6, representing a standard non-selective surface flat plate solar collector, the critical Rayleigh number is increased 11% (ie convection suppression is enhanced) by giving one of the surface a low emissivity,
(\(\varepsilon_p = .9, \varepsilon_p = .065, \text{Fig. 4.5}\)) representing a collector having a selective surface. If both bounding surfaces are given low emissivities (\(\varepsilon_p = \varepsilon_p = .065, \text{Fig. 4.4}\)) a further increase in the critical Rayleigh number of 26\% is observed. The above comparisons are for the results having \(\theta \leq 75^\circ\). For \(\theta = 90^\circ\) the same trend is observed.

4.6 Correlation Equations

4.6.1 Critical Rayleigh Numbers, \(\theta < 75^\circ\)

Table 4.2 shows the value of the critical Rayleigh number, \(R_{ac}\), for each experimental set up, evaluated using the data at \(\theta = 0^\circ\). As mentioned earlier, the critical Rayleigh number is that Rayleigh number at which the Nusselt number significantly departs from unity, indicating departure from a conductive state. The values in Table 4.2 were obtained by fairing a curve through those data points having \(Nu > 1.2\) and extrapolating that curve to \(Nu = 1\); the corresponding value of \(Ra\) is \(R_{ac}\).

Properly designed honeycombs in solar collectors have \(Ra < R_{ac}\) over the full design operating range. Hence it is important that \(R_{ac}\) be predictable.

Based on the analytical methods developed and tabulated by Sun [31], Buchberg et al [32] gave an approximate equation for \(R_{ac}\) at \(\theta = 0^\circ\) which, for the present instance of large \(A_p\), can be made to reduce to:

\[
R_{ac} = A_E^2 \left[ 768 + \frac{1536 A_E}{C} + 48 \varepsilon N A_E \cdot \frac{(1-S)}{T-S(T-\varepsilon)} \right]
\]
The quantity $S$ in this expression is the value of a definite integral containing $A_p$ and $A_E$ as parameters. Although graphed in [32] a more satisfactory representation is a fitted functional relation. For the case $A_p \to \infty$ such a fitted function was found in the present study to be:

$$S = 1.0102 - \frac{1.4388}{A_E} - \frac{9.4653}{A_E^2} + \frac{31.440}{A_E^3} - \frac{27.515}{A_E^4}$$

(4.2)

This formula is valid for $A_E \geq 2$ to within about 1%. Values of $R_a_c$ calculated from equations 4.1 and 4.2 are tabulated on Table 4.2 along with the measured values. The method due to Sun and Buchberg et al just outlined cannot be fully predictive since it does not encorporate the effect of $\epsilon_{p1}$ and $\epsilon_{p2}$. However, notwithstanding this, the comparison is seen to be quite good. The maximum deviation occurs when $\epsilon_{p1} = \epsilon_{p2} = .9$, being 34% in this case. Aside from this case, predictions are within 21%.

The preceding discussion has been restricted to the case $\theta = 0$. For other angles, inspection of Figures 4-2 to 4-7 shows that the value of $R_a_c \cos \theta$ is approximately independent of $\theta$, provided $\theta \leq 75^\circ$.

Hence the value of $R_a_c$ for other angles can be found from $R_a_c$ at $\theta = 0$, as given by equation 4.1, by division by $\cos \theta$, ie:

$$R_a_c = \frac{A_E^2}{\cos \theta} \left[ 768 + \frac{1536A_E}{C} + 48 \epsilon \frac{NA_E}{C} \cdot \frac{1 - S}{1 - S(1 - \epsilon)} \right]$$

(4.3)
<table>
<thead>
<tr>
<th>Expt'1 Setup</th>
<th>$A_e$</th>
<th>$\varepsilon$</th>
<th>$\varepsilon_{p1}$</th>
<th>$\varepsilon_{p2}$</th>
<th>Measured $Ra_C$</th>
<th>Predicted $Ra_C$ (Eqn 4.3)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>27,500</td>
<td>30,500</td>
<td>+11</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>204,000</td>
<td>206,000</td>
<td>+1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>8,660</td>
<td>8,780</td>
<td>+1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>.9</td>
<td>.065</td>
<td>.065</td>
<td>65,200</td>
<td>62,500</td>
<td>-4.3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>.9</td>
<td>.9</td>
<td>.065</td>
<td>51,800</td>
<td>62,500</td>
<td>+21</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>.9</td>
<td>.9</td>
<td>.9</td>
<td>46,500</td>
<td>62,500</td>
<td>+34</td>
</tr>
</tbody>
</table>
Equation 4.2 and 4.3 are the recommended equations for determining critical Rayleigh numbers for horizontal slit honeycombs.

4.6.2 Correlation Equation for Nu for $\theta \leq 75^0$

A correlation equation for Nu as a function of the many variables on which it depends so as to fit all the data was sought. A form similar to that used by Hollands [13] to represent data at $\theta = 0$ for square-celled honeycombs was used. The fact that plotting the data against $Ra \cos \theta$ rather than $Ra$ caused the data at different angles to fall roughly on a single curve was also used in choosing the equation form. The chosen equation is:

$$Nu = 1 + c(Ra \cos \theta)^{1/3} [1 - \exp \{- (a + b \theta) [\left(\frac{Ra}{Ra_c}\right)^n - 1]\}]$$  \hspace{1cm} (4.4)$$

This equation is to be used for $Ra > Ra_c$, $0 \leq \theta \leq 75^0$. For $Ra < Ra_c$ the fluid is virtually in a conductive state so that:

$$Nu = 1$$  \hspace{1cm} (4.5)$$

The value of $Ra_c$ in equation 4.4 is determined from equation 4.3. The quantities $a$, $b$, $c$ and $n$ are assumed to be constants. There values were chosen so as to minimize the sums of the square of the departure of the measured $Nu$ from that given by equation 4.4 over all (347) data points having $\theta \leq 75^0$. The resulting values for the constants were:
\[ a = 0.18 \]
\[ b = 1.2 \times 10^{-3} \text{(degree}^{-1}) \] (4.6)
\[ c = 0.131 \]
\[ n = 0.513 \]

The average rms error in Nu associated with this correlation equation is 18%. The dependence of Nu on \( A_E \), \( C \), \( \varepsilon \) and \( N \) is represented entirely through \( \text{Ra}_c \), as given by equations 4.3 and 4.2.

Equations 4.2 to 4.6 are the recommended equations for determining Nu for \( \theta \leq 75^\circ \). Their range of validity on Ra is: \( \text{Ra} < 100 \text{Ra}_c \).

4.6.3 Correlation Equation for Nu for \( \theta = 90^\circ \)

Inspection of Figures 4.2 to 4.7 shows that quite a different functional dependence of Nu on Ra is required for \( \theta = 90^\circ \), from that for \( \theta \leq 75^\circ \). An analysis was performed on this problem which yielded an approximate expression for the expected form of the Nu-Ra relation. The expression is:

\[ \nu = 1 + \frac{(1 + 2 \XY) - \sqrt{1 + 4 \XY}}{2XY^2} \] (4.7a)

where:
\[ X = \gamma R a^2 / A_E \] (4.7b)
\[ Y = (0.306(RaA_E)^{1/5} - 1)^{-1} \] (4.7c)

The value of \( \gamma \) depends on the effect of wall conduction and radiation on the free convection, i.e., on the parameters \( C, N, \varepsilon, \varepsilon_{p1}, \varepsilon_{p2} \). For \( C = 0 \) its analytical value is known to be \( \gamma = 0.824 \times 10^{-6} \). If \( \varepsilon N = 0 \)
and $C = \infty$ its value is expected to be $\gamma = 2.756 \times 10^{-6}$. These two values for $\gamma$ do not necessarily establish bounds on $\gamma$ but do give its expected order of magnitude. Values of $\gamma$ were determined, for each of the experimental set ups, by choosing that value which minimizes the rms deviation of the measured Nusselt number data from that calculated from equation 4.7 at the same $Ra$. All data having $Nu \leq 3$ were included. The rms error on the fits was of order of 4%. The resulting values for $\gamma$ are given in Table 4.3. Significant dependences of $\gamma$ on $\epsilon$, $\epsilon_{p1}$, $\epsilon_{p2}$ and $A_E$ are to be noted. For experimental setups 1, 2 and 3 a monotonic increase of $\gamma$ with $A_E$ is noted. A strong effect of $\epsilon_{p2}$ is also to be noted. No fit of $\gamma$ as a function of $A_E$, $\epsilon$, $\epsilon_{p1}$, $\epsilon_{p2}$ has been attempted. Rather designers are advised to interpolate directly from Table 3.2 for a given set of $A_E$, $\epsilon$, $\epsilon_{p1}$, and $\epsilon_{p2}$. Alternately he may use universally the average value of $\gamma$ for all honeycombs tested which is: $\gamma = 0.961 \times 10^{-6}$ and accept the corresponding error in $Nu$. An estimate of this error indicates that it can be as high as 40% in $Nu$ but over most of the range it should be closer to 20% or less. The error in this procedure is greatest at $A_E = 10$. Further research, both analytical and experimental is required to better evaluate the functional dependance of $\gamma$.

Figure 4.10 shows the plot of the data at $90^\circ$ as compared to the correlation equation 4.7 using the values of $\gamma$ given in Table 4.3. A good fit is observed up to values of $Nu$ of approximately 5. The data
Comparison of data at $\theta = 90^\circ$ with correlation equation 4-7. Values of $\gamma$ and $\eta$ used in this equation are given in Table 4.3. (See Table 4-1 for full details of each experimental set-up.)
Table 4.3

Values of $\gamma$ for the Different Experimental Set Ups

<table>
<thead>
<tr>
<th>Expt'1 Setup No.</th>
<th>$A_E$</th>
<th>$\varepsilon$</th>
<th>$\varepsilon_{p1}$</th>
<th>$\varepsilon_{p2}$</th>
<th>$\gamma \times 10^6$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>.600</td>
<td>.094</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>1.172</td>
<td>.363</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>.562</td>
<td>.013</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>.9</td>
<td>.065</td>
<td>.065</td>
<td>.560</td>
<td>.003</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>.9</td>
<td>.9</td>
<td>.065</td>
<td>.486</td>
<td>.303</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>.9</td>
<td>.9</td>
<td>.9</td>
<td>2.385</td>
<td>-.075</td>
</tr>
<tr>
<td>Avg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.961</td>
<td></td>
</tr>
</tbody>
</table>
show a departure of Nu from unity at small Ra, in some cases. The explanation for this effect is not known with confidence at the present time although it is expected to be due to convection driven by large local gradients in wall temperature enforced by radiative transfer near the bounding plates. These gradients can generate small convection currents which augment the convective heat transfer. To account for this effect in the fitting of the correlation equation, a term \( \eta \) was added as an additive constant on Nu:

\[
Nu = 1 + \eta + \frac{1 + 2xy - \sqrt{1 + 4xy}}{2xy^2} \tag{4.8}
\]

\( \eta \) was chosen for each experimental set up in the same way as for \( \gamma \), and in fact simultaneously with the evaluation of \( \gamma \). Values of \( \eta \) are given in Table 4.3. No attempt has been made to fit \( \eta \) to the relevant variable since no consistent trend is evident and its effect on Nu is not overly large. The average \( \eta \) is .12 and designers may wish to incorporate this value when evaluating Nu.

Designers of horizontal slit honeycombs may wish to design the honeycomb so as to effectively suppress free convection over the operating range. As was suggested in [1], effective suppression has probably been achieved provided \( Nu \leq 1.2 \). Substituting a value of \( Nu = 1.2 \) into equation 4.7 yields an implicit expression for the corresponding Ra number, which will be denoted by \( Ra_s \). An explicit equation for \( Ra_s \)
has been obtained from this equation by introducing some minor approximations. The equation is:

\[
Ra_s = \frac{A_E^{4}(0.2 + 0.245/A_E^{1.39})^{1/2}}{\gamma^{1/2}}
\]  

(4-9)

4.7 Application to Design of Honeycomb Collectors

The application of the preceding correlation equations to the design of honeycomb solar collectors will now be outlined by means of an example. Suppose it is required to design a horizontal slit honeycomb for a solar collector using a plastic film having \( \varepsilon = 0.4 \), \( \delta = 0.001 \) inch and \( k_w = 0.3 \) Btu/fthrF. The following conditions are assumed to apply: mean absorber plate temperature = \( T_h = 160 \) F; mean glass cover temperature = \( T_c = 40 \) F; angle of inclination of collector, \( \theta = 60^\circ \). The problem is to find permissible values of \( L \) and \( L_{NS} \) so as to ensure suppression of free convection.

Buchberg et al.'s modified equation, namely \((4.3)^\dagger\), will be used with \( S \) evaluated from \((4.2)\). The problem will be solved in British units. We have:

\[
T_m = \frac{160 + 40}{2} = 100 \text{ F} = 560 \text{ R}
\]

\[
\Delta T = 160 - 40 = 120 \text{ F}
\]

At \( T_m = 100 \) F, \( k_f = 0.0154 \) Btu/fthrF and \( g \beta/(\nu \alpha) = 1.267 \times 10^6 \text{ F}^{-1} \text{ ft}^{-3} \) or 733 \text{ F}^{-1} \text{ inch}^{-3} \. Consequently:

\(\dagger\) If \( \theta \) were in range: \( 80^\circ \leq \theta \leq 90^\circ \), equation (4-9) would have been used.
Ra = \frac{g \beta AT L^3}{\nu \alpha} = 87960 \ L^3

where \ L is expressed in inch. Also

\[ C = \frac{k_f L}{k_w L} = \frac{.0154 \times L/12}{.3 \times .001/12} = 51L \]

and

\[ N = \frac{4\sigma T m^3 L}{k_f} = \frac{4 \times .1713 \times 10^{-8} \times 560^3 \times L}{12 \times .0154} = 6.51L \]

where again \ L is in inch. Substitute these values into equation (4.3):

\[
87960L^3 = \frac{A_E^2}{\cos 60^\circ} \left[ 768 + \frac{1536A_E + 48 \times 4 \times 6.51 \times L x A_E}{51L} \right]
\]

where

\[ \tilde{A}_E = A_E \left( \frac{1-S}{1-0.6S} \right) \]

where \ S is given in terms of \ A_E by equation 4.2. The above equation can be expressed as:

\[
\left( \frac{57.26}{A_E^2} \right) L^4 - 0.1625 \tilde{A}_E L^2 - L - 0.039 A_E = 0
\]

- an equation which can be solved for \ L for various values of \ A_E. Solutions are given in Table 4-4. For each \ L, the corresponding value of \ L_{NS} in the table is the minimum value of \ L_{NS} required to suppress convection.

- i.e. larger values of \ L_{NS} will experience convection. (Conversely for each \ L_{NS}, the corresponding value of \ L in the table is the maximum permissible \ L in order to ensure suppression.)
Table 4-4

Permissible Values of L and \( L_{NS} \)
for Suppression of Convection for
Set Conditions

<table>
<thead>
<tr>
<th>( A_E )</th>
<th>L (inch)</th>
<th>( L_{NS} ) (inch)</th>
<th>( h = k_f/L ) Btu/ft(^2)hrF</th>
<th>Minimum value of ( h ) if no honeycomb Btu/ft(^2)hrF</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>.610</td>
<td>.203</td>
<td>.303</td>
<td>.54</td>
</tr>
<tr>
<td>4</td>
<td>.762</td>
<td>.190</td>
<td>.243</td>
<td>.54</td>
</tr>
<tr>
<td>5</td>
<td>.907</td>
<td>.181</td>
<td>.204</td>
<td>.54</td>
</tr>
<tr>
<td>6</td>
<td>1.048</td>
<td>.175</td>
<td>.176</td>
<td>.54</td>
</tr>
<tr>
<td>7</td>
<td>1.184</td>
<td>.169</td>
<td>.156</td>
<td>.54</td>
</tr>
<tr>
<td>8</td>
<td>1.32</td>
<td>.165</td>
<td>.14</td>
<td>.54</td>
</tr>
<tr>
<td>9</td>
<td>1.45</td>
<td>.161</td>
<td>.127</td>
<td>.54</td>
</tr>
<tr>
<td>10</td>
<td>1.57</td>
<td>.157</td>
<td>.118</td>
<td>.54</td>
</tr>
</tbody>
</table>
Also tabulated in Table 4-4 is the free convective heat transfer coefficient corresponding to each $L$. (Since convection is suppressed, $\text{Nu} = 1$ and $h = \frac{k_f}{L}$.) For the case of no honeycomb the minimum value of $h$ occurs when $L = 0.339$ inch [1], and is given by $h = 0.54$ Btu/ft$^2$hr$^°$F. Thus reductions in $h$ by factors ranging from $0.54/0.303 = 1.8$ for $A_E = 3$ to $0.54/0.118 = 5.6$ for $A_E = 10$ should be possible by using horizontal slit honeycombs in this instance.

4.8 A Note on Radiative Transfer Across Honeycombs

In order to establish the overall reduction in heat transfer due to the honeycomb the radiative component must be taken into account as well. Generally it is considered that the honeycomb suppresses long-wave radiant heat transfer as well as convective transfer (see, for example, [10], so that an overall reduction in heat loss due to the insertion of a honeycomb in an air layer is always ensured.$^\dagger$ However, there is now evidence that, although this may be true if $\epsilon_{p1}$ and $\epsilon_{p2}$ are near unity, for other values of $\epsilon_{p1}$ and $\epsilon_{p2}$ an increase in radiative exchange may be introduced by the honeycomb. Table 4-5 presents values of the radiant exchange factor $F$ measured in the present experiments, where $F$ is defined by:

$$F = \frac{q_r}{\sigma(T_h^4 - T_c^4)} = \frac{h_r(T_h - T_c)}{\sigma(T_h^4 - T_c^4)}$$

where $q_r$ is the radiant flux across the honeycomb, calculated from

$^\dagger$Provided, of course, that the honeycomb is properly designed so as to suppress convection.
Table 4-5

Table of values of $F$
found in present experiments

<table>
<thead>
<tr>
<th>Expt'1 Set up no</th>
<th>$A_E$</th>
<th>$\varepsilon$</th>
<th>$\varepsilon_{pl}$</th>
<th>$\varepsilon_{p2}$</th>
<th>$F$</th>
<th>Value of $F$ if no honeycomb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>.066</td>
<td>.033</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>.086</td>
<td>.033</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>.13</td>
<td>.065</td>
<td>.065</td>
<td>.049</td>
<td>.033</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>.9</td>
<td>.065</td>
<td>.065</td>
<td>.134</td>
<td>.033</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>.9</td>
<td>.9</td>
<td>.065</td>
<td>.22</td>
<td>.064</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>.9</td>
<td>.9</td>
<td>.9</td>
<td>.41</td>
<td>.82</td>
</tr>
</tbody>
</table>
where \( q_T \) is the measured total heat transfer across the honeycomb. The quantity \( h_r = q_r/(T_h - T_c) \) is the radiant heat transfer coefficient. For no honeycomb \( F \) is given by:

\[
F = (\epsilon_{p1}^{-1} + \epsilon_{p2}^{-1} - 1)^{-1}
\]

Values of \( F \) for no honeycomb corresponding to the \( \epsilon_{p1} \) and \( \epsilon_{p2} \) for each experimental setup are also given in the Table. In all cases except No. 6 the radiative heat transfer coefficient has been increased by insertion of the honeycomb. The reason for the increase is speculated to be due to the interaction of the radiative mode of heat transfer across the honeycomb with that due to conduction across the stagnant air. Previous theories have hypothesized that these two modes can be calculated separately and independently. However if either or both of \( \epsilon_{p1} \) and \( \epsilon_{p2} \) is low, the temperature distribution imposed on the honeycomb wall by the radiant transfer substantially increases the temperature gradient of the honeycomb wall from the linear gradient expected from purely conductive considerations. The air adjacent to the wall is forced to follow this distribution, thereby increasing the conductive mode. The increased heat transfer shown in the table is therefore most probably due to air conduction, rather than radiation. (Its appearance in a radiative term is due to calculating \( q_r \) according to 4-10, which implicitly assumes that both

\[\text{near the bounding surfaces}\]
the air and the honeycomb wall have one-dimensional, linear temperature
distributions.) It should be noted that this radiant-conduction coupling
effect can be expected to increase with decreasing $L_{NS}$ so that the measured
values of $F$ shown in Table 4-5, which are based upon $L_{NS} = .5$ inch, may
be lower bounds for those expected when $L_{NS}$ has values more closely allied
to those which must be used in practice - such as those in Table 4-4.

To obtain an indication of the effect on the overall heat
transfer across the honeycomb, of this increased value of $F$ due to radiant-
conduction coupling, values of the overall honeycomb heat loss coeffi-
cient, $U_t = h_r + h + h_w$ have been calculated assuming the values of $F$ of
Table 4-5 applied to the honeycombs of Table 4-4. (In fact they will
not because the values of $\varepsilon$ and $L_{NS}$ are not matched.) These values are
then compared to the corresponding value without a honeycomb. The re-
sults are shown in Table 4-6. Reductions in overall heat transfer are
experienced in all cases. However more work is required to fully est-
ablish the predictability of $F$ in honeycomb situations.
Table 4-6

Calculated overall heat loss coefficients
for honeycomb and non-honeycomb situations

<table>
<thead>
<tr>
<th>$A_e$</th>
<th>$L$</th>
<th>$L_{NS}$</th>
<th>$\varepsilon_p1$</th>
<th>$\varepsilon_p2$</th>
<th>$U_t$, Btu/ft$^2$hr$^\circ$F</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>.907</td>
<td>.181</td>
<td>.065</td>
<td>.065</td>
<td>.328</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.580</td>
</tr>
<tr>
<td>10</td>
<td>1.57</td>
<td>.157</td>
<td>.065</td>
<td>.065</td>
<td>.252</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>.580</td>
</tr>
<tr>
<td>3</td>
<td>.610</td>
<td>.203</td>
<td>.065</td>
<td>.065</td>
<td>.421</td>
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<td></td>
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<td>.580</td>
</tr>
<tr>
<td>5</td>
<td>.907</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.52</td>
</tr>
</tbody>
</table>

67.
5. CONCLUSIONS

1. Equation (3-7) is the recommended equation for evaluating free convective heat transfer across the air layer bounded by a V-corrugated and a flat plate. The equation is valid over the range $10^3 \leq Ra \leq 10^6$, $0 \leq \theta \leq 60^\circ$, $1 \leq A \leq 4$ except in the range $10^3 < Ra < 10^4$ for $A \sim 1$. In this region Figure 3-4 should be used directly.

2. In order to best evaluate the optimum value of $L$ and $H$ for a V-corrugated absorber plate solar collector, the designer is advised to plot a graph such as Figure 3-7. For the rather typical set of circumstances: $\theta = 45^\circ$, $T_c = 15.5^\circ C$, $T_h = 71^\circ C$, the optimum values of $L$ and $H$ are about 1.5 and 1.25 cm respectively. Under these conditions the free convective heat loss will be about 20% greater than that for a flat absorber plate set at its optimum $L$.

3. As a general guide to free convective heat loss from roughened absorber plates, provided the ratio of the height of the protrusions to the average spacing between the plate and the plane glass cover is less than about .25, they should have little effect on the free convective heat transfer, provided $Ra < \sim 10^5$.

4. Horizontal slit honeycombs show greater convection suppression capabilities than square-celled for the same amount of honeycomb film material, for angles of tilt of $60^\circ$ and higher. This means that the same convection suppression capabilities can be achieved with less honeycomb material. However for angles of tilt less than about $45^\circ$, the convection suppression capabilities of horizontal slits are
significantly less than that of square celled honeycombs.

5. The bounding plate emissivities, $\varepsilon_{p1}$ and $\varepsilon_{p2}$ can decrease by up to 35% the critical Rayleigh governing the first appearance of free convective, cellular motion in honeycombs.

6. The expression recommended by Buchberg et al, for predicting the critical Rayleigh number in horizontal honeycombs, can be made to apply to inclined honeycombs, by division by $\cos \theta$, for $\theta$ up to 75°, provided the plan aspect ratio, $A_p$ is large as is the case for horizontal slits. (Earlier work had shown that for $A_p = 1$ such as scaling law did not apply.) For $75^\circ \leq \theta \leq 90^\circ$, equation (4-9) is recommended for finding the effective critical Rayleigh number. Calculations based on a typical set of conditions indicate that in order to achieve convection suppression with horizontal slits at $\theta = 60^\circ$, the aspect ratio $A_e$ must be greater than 2 and the cell width must be of the order of 3/16 inch or .5 cm. Sample calculations are given in this report for calculating conditions for suppression for other sets of conditions.
6. LIST OF REFERENCES


34. Smart, D.R. "Convection Suppression and Heat Transfer in Air Constrained by Slit Type Honeycomb with Application to Flat Plate Solar Collector Design", submitted as an M.A. Sc. Thesis, Department of Mechanical Engineering, University of Waterloo, Ontario, Canada (1978).


36. Ibid, To be published in Journal of Heat Transfer, ASME.
7. NOMENCLATURE

a  correlation constant, \( a = 0.18 \) (see equation (4.6))
A  depth ratio, \( A = L/H \)
\( A_E, A_p \)  honeycomb aspect ratios - see Figure 4-1
b  correlation constant \( b = 1.2 \times 10^{-3} \) (degree\(^{-1}\)) (see equation 4-6)
B  correlation function given by equation (3-10)
c  correlation constant \( c = 0.131 \) (see equation (4-6))
C  \( C = k_fL/(k_wL) \), dimensionless
F  \( F = q_r/(\sigma(T_h^4 - T_c^4)) \)
g  acceleration of gravity
h  free convective heat transfer coefficient between surface at \( T_h \) and surface at \( T_c \); in the case of section 3, it is based on unit area of flat surface.
h_r  radiative heat transfer coefficient across honeycomb, \( h_r = q_r/(T_h - T_c) \)
h_w  heat transfer coefficient across honeycomb due to honeycomb wall conduction, based on same area as \( h \)
H  in section 3; \( H \) is the height of the V's (see Figure 3-1)
in section 4; \( H = k_f\delta/(k_wL) \)
k, \( k_f \)  thermal conductivity of air
\( k_w \)  thermal conductivity of honeycomb wall material
K  correlation function given by equation (3-9)
L  average spacing between surface at \( T_h \) and surface at \( T_c \), see Figures (3-1) and (4-1)
\( L_{NS} \)  spacing, measured in the upslope direction, between rectangular honeycomb walls (see Figure 4-1)
L_{EW} \quad \text{spacing measured in the transverse direction, between rectangular honeycomb walls, (see Figure 4-1)}

n \quad \text{correlation constant, } = .513, \text{ (see equation 4-6)}

N = 4 \sigma T_m^3 L/k_f \quad \text{Nusselt number, } = hL/k

Nu \quad \text{Nusselt number in the conduction regime}

\text{Nu}_c \quad \text{Nusselt number in the conduction regime}

\text{Pr} \quad \text{Prandtl number } = \nu/\alpha \text{ or } \nu/\lambda

q \quad \text{free convective heat flux across air layer (includes conduction through the gas)}

q_c \quad \text{purely conductive heat flux across air layer}

q_r \quad \text{radiant flux across air layer or honeycomb}

\text{Ra} \quad \text{Rayleigh number, } = g\beta AT L^3/(\nu \lambda), \text{ or } g\beta AT L^3/(\nu \alpha)

\text{Ra}_c \quad \text{critical Rayleigh number - i.e. value of Ra at which convection initiates}

\text{Ra}_t \quad \text{correlation function given by equation (3-11)}

\text{S} \quad \text{a definite integral due to Sun [31]; given as a function of } A_E \text{ in equation (4-2), provided } A_p = \infty

T_c, T_h \quad \text{temperature of cold and hot bounding surfaces to air layer, respectively}

T_m = (T_h + T_c)/2

\Delta T = T_h - T_c

W \quad \text{length of air layer in upslope direction (see Figure 3-1)}

U_t = h_r + h + h_w
Greek Letters

\( \alpha \)  thermal diffusivity of air

\( \beta \)  volumetric expansion coefficient of air

\( \gamma \)  correlation quantity; measured values are given in Table 4.3

\( \delta \)  semi-thickness of walls forming the honeycomb, - i.e. of polyethylene sheet.

\( \epsilon \)  emissivity of honeycomb wall

\( \epsilon_{p1}, \epsilon_{p2} \)  emissivity of plates bounding the honeycomb on top and bottom face,
(not necessarily respectively; the order is immaterial provided

\[ \frac{T_h}{T_c} \approx 1. \])

\( \theta \)  angle of inclination of the air layer from horizontal, measured in degrees

\( \lambda \)  thermal diffusivity of air

\( \nu \)  kinematic viscosity of air

\( \eta \)  correlation constant; see Table 4.3

\( \sigma \)  Stefan Boltzmann constant
8. LIST OF CONTRIBUTING PERSONNEL

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Undergraduate Students:
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9. CONTRACT COMPLIANCE

On April 29, 1976 ERDA granted additional funds to the University of Waterloo for support of Phase II of the project "Methods for Reducing Heat Losses from Flat Plate Solar Collectors", as described in Waterloo Research Institute proposal No. 510-08 dated Oct. 23, 1975. (The Grant Number at that time was E(11-1) - 2597 (Formerly NSF Grant No. AER 74-09110), Amendment No. 1.) This report contains full compliance to the work scope as contained in that proposal. The project was under the direction of K.G.T. Hollands, Department of Mechanical Engineering.

Publications and conference papers arising out of this research grant (Phase 2) are listed as references [35] and [36] in the List of References of this report. Additional papers are in preparation for the ISES-American Section Meeting, Denver, August 27-31, 1978 and the ASME Winter Annual, San Francisco, Dec. 10-15, 1978. Publications arising out of Phase 1 are listed as Reference Numbers [2], [3], [7], and [11].